N = impeller rotational sped, s^{-1}
N_A = aeration number, Q/ND^3, dimensionless
N_B = number of impeller blades
N_{Fr} = Froude number, N^2D/g, dimensionless
N_P = power number, $P_o g_c/\rho_L N^3 D^5$, dimensionless
N_{Re} = impeller Reynolds number, $D^2 N \rho_L/\mu$, dimensionless
N_{We} = impeller Weber number, $N^2 D^3 \rho_L/\sigma$, dimensionless
P_g = mechanical agitation power in gas-liquid dispersion, W
P_o = mechanical agitation power in ungassed liquid, W (Kg m/s in N_P)
P_q = sparge gas isothermal expansion power ($P_q = \rho_L g h_L Q$), W
Q = volumetric gas sparging rate (30°C, $1.013 \cdot 10^5$ N/m^2), m^3/s
T = tank internal diameter, m
T_I = impeller blade thickness, m
V = liquid volume, m^3
W_B = baffle width, m
W_I = impeller blade width, m
z = exponent in Equation (2), dimensionless
ϕ = gas holdup volume fraction, dimensionless
μ = liquid phase viscosity, $N \cdot s/m^2$
ρ = mass density, kg/m^3
σ = air-liquid surface tension, N/m

Subscripts

D = property of gas-liquid dispersion
L = property of liquid

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Degrees of Anisotropy for Fluid Flow and Diffusion (Electrical Conduction) Through Anisotropic Porous Media

GRAHAM NEALE

Chemical Engineering Department
University of Ottawa
Ottawa, Ontario, Canada

This work involves a quantitative comparison between diffusion (or electrical conduction) and fluid flow occurring in the principal directions of certain simple anisotropic porous media, namely, clusters of parallel circular (or elliptic) cylindrical fibers. The degrees to which the observed macroscopic properties, permeability, and effective diffusivity (electrical conductivity) depend upon the direction of flow are determined analytically. Systems of parallel fibers are shown to be considerably more anisotropic with respect to fluid flow than with respect to diffusion (or electrical conduction), this effect being very pronounced with flattish fibers. It would not appear that any general analogy exists between diffusion (electrical conduction) and fluid flow occurring within anisotropic porous media.

SCOPE

Many porous media encountered in practice, especially those formed as a result of industrial or geological sedimentation processes, are anisotropic in constitution, for example, filter beds, river beds, drilling mud cakes, underground rock formations, and oil sands. That is to say, their measured macroscopic transport properties, such as permeability, diffusivity, and electrical conductivity, are direction dependent. Anisotropy is generally a consequence of the preferential orientation and asymmetric geometry of the grains which constitute the porous medium (Rice et al., 1970; Rees, 1968). Anisotropy can also occur as a direct result of a spatially varying porosity in a system, but this particular category of anisotropic media is not considered in this exploratory analysis.

It is well known that anisotropy in porous media is a difficult property to characterize quantitatively (Rice et al., 1970). The quantity referred to as "degree of anisotropy" by Rice et al. in their definitive experimental study and review of fluid flow through anisotropic porous media is generally understood to reflect the extent to which the measured macroscopic transport properties of a particular medium depend upon the direction of measurement. The objectives of the present work are to establish analytically that this degree of anisotropy (D.O.A.) is a function not only of the structural properties of the porous medium but also of the particular transport process being studied, and to determine if there is any simple relation between the D.O.A.'s for fluid flow and diffusion (electrical conduction). These important aspects concerning anisotropic porous media have received surprisingly little attention in the literature to date, despite the fact that an improved understanding of transport processes occurring in anisotropic porous media is definitely required in several areas of petroleum reservoir engineering and, more important, in petroleum exploration well-logging operations (Pirson, 1963).

The present work involves a quantitative analytical comparison between the D.O.A. of certain simple homogeneous anisotropic porous media for creeping Newtonian fluid flow and simple diffusion (or electrical conduction). The term homogeneous is here taken to mean that the local porosity is uniform throughout the medium. The case of nonhomogeneous (nonuniform porosity) anisotropic media is not considered in this work.

CONCLUSIONS AND SIGNIFICANCE

The degrees of anisotropy (D.O.A.) of certain simple anisotropic porous media which permit transport in any arbitrary direction, namely, clusters of parallel circular (or elliptic) cylindrical fibers, are calculated for both fluid flow and diffusion (electrical conduction). The predictions for fluid flow have been taken directly from the literature (Happel, 1959; Kuwabara, 1959; Epstein and Masliyah, 1972), while the predictions for diffusion are based upon the generalized geometric model developed by Neale and Nader (1973, 1974) for a bed of spherical particles and extended recently to circular cylinders (Neale and Masliyah, 1975) and spheroidal particles (Neale and Nader, 1976).

Predictions of the D.O.A. are presented for both fluid flow and diffusion (electrical conduction) through clusters of circular (or elliptic) cylindrical fibers as functions of the porosity ϵ (see Figures 2, 5, and 6). It has been concluded that there exists no simple general relationship between the D.O.A.'s for fluid flow and diffusion, indicating that there is no direct analogy between these basic transport processes when they occur in anisotropic porous media. The presented results (Figure 6) indicate that the D.O.A.'s for diffusion and fluid flow can differ appreciably in magnitude, thereby confirming that the quantity degree of anisotropy is intrinsically an ambiguous parameter, unless it is qualified by reference to a particular transport process.

The predictions and general conclusions presented here should find useful application in the numerous areas where anisotropic porous media are encountered in practice. Notable examples of anisotropic media include filter cakes, sewage sediments, river beds, petroleum reservoir rock formations, and, among others, the drilling muds and mud cakes employed during the drilling of oil wells.

It is hoped that the conclusions reached in this preliminary study will stimulate further research into transport processes occurring within anisotropic porous media, an area which, although of significant practical importance nowadays, has received inadequate attention in the literature.

GENERAL CONSIDERATIONS

Consider an arbitrary homogeneous (uniform porosity) anisotropic porous medium. If a Cartesian frame of reference (x, y, z) is selected such that its axes are collinear with the three mutually orthogonal principal axes of the anisotropic porous medium, assuming they exist, then the matrices associated with the permeability and diffusivity tensors become diagonalized (Rice et al., 1970; Neale and Nader, 1976), namely

$$\begin{bmatrix} k_x & O & O \\ O & k_y & O \\ O & O & k_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} D_x & O & O \\ O & D_y & O \\ O & O & D_z \end{bmatrix}$$

respectively. If the three elements in each of the above matrices are known quantitatively, then the anisotropy of the medium is fully specified with respect to fluid flow and diffusion (or electrical conduction).

Fortunately, in many practical situations involving homogeneous anisotropic porous media, especially those formed during sedimentation processes, the macroscopic properties in the two principal directions which are parallel with the bedding plane will be equal, thereby greatly simplifying the general form of the above matrices. For example, in a horizontal filter cake which has been formed as a result of sedimentation in the vertical z direction, there is no statistical reason to presuppose that $k_x \neq k_y$ or $D_x \neq D_y$.

We now turn our attention to some very simple anisotropic porous media which permit transport in any arbitrary direction and which lend themselves to a theoretical analysis for both fluid flow and diffusion (electrical conduction).

POROUS MEDIA COMPOSED OF PARALLEL CIRCULAR CYLINDRICAL FIBERS

Perhaps the simplest homogeneous anisotropic porous medium which permits transport in any arbitrary direction is a cluster of long parallel circular cylindrical fibers of equal radius, distributed in a random array (Figure 1). If the frame of reference (x, y, z) is chosen such that

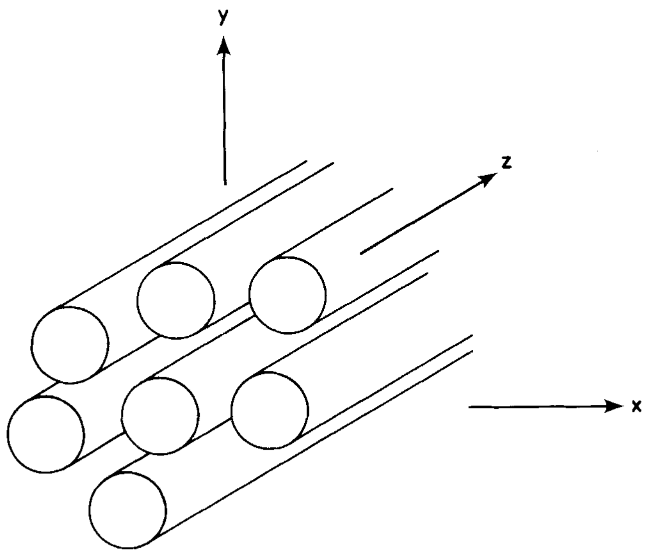

Fig. 1. A bank of parallel monosized circular cylinders.

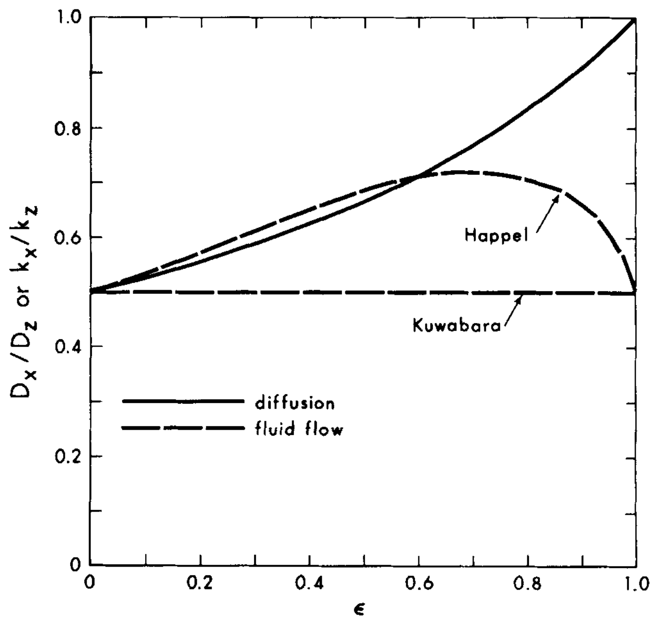

Fig. 2. Comparison between D_x/D_z [Equation (3)] and k_x/k_z [Equations (7) and (8)] for parallel monosized circular cylinders. ———, Neale and Nader (1976); — — — —, Happel (1959) and Kuwabara (1959).

the z direction is collinear with the axes of the cylinders, then it is clear from considerations of symmetry that the macroscopic properties in any direction orthogonal to the z direction must be equal.

For the case of diffusion (electrical conduction),* predictions for the effective macroscopic diffusivities in the three principal directions are available from an earlier study involving spheroidal particles (Neale and Nader, 1976):

$$D_x/D = D_y/D = \epsilon/(2 - \epsilon) \tag{1}$$

$$D_z/D = \epsilon \tag{2}$$

where D denotes the absolute diffusivity in the unobstructed fluid. Equation (1) actually constitutes Maxwell's classic result and therefore represents the exact solution for high porosities ($\epsilon \to 1.0$), while Equation (2) is exact for all physically attainable values of ϵ (since the cross-sectional area available for diffusion does not change in the z direction). Moreover, for $\epsilon > 0.5$, the predictions of Equation (1) are virtually indistinguishable from those of Lord Rayleigh's (1892) exact series solution for parallel monosized circular cylinders in a square array. For the present system, we define a characteristic D.O.A. with respect to diffusion as D_x/D_z (a D.O.A. is defined here in such a manner that its magnitude is less than unity). Thus, from Equations (1) and (2)

$$D_x/D_z = 1/(2 - \epsilon) \tag{3}$$

This prediction is displayed in Figure 2.

For the case of creeping Newtonian fluid flow, the permeabilities in the principal directions have been calculated by Happel (1959) using his well-established free-surface model:

$$k_x = k_y = \frac{-R^2}{8(1-\epsilon)}\left[\ln(1-\epsilon) + \frac{\epsilon(2-\epsilon)}{\epsilon^2 - 2\epsilon + 2}\right] \tag{4}$$

$$k_z = \frac{-R^2}{8(1-\epsilon)}\,[2\ln(1-\epsilon) + 2\epsilon + \epsilon^2] \tag{5}$$

Alternative predictions were developed, independently, by Kuwabara (1959) using his analogous zero-vorticity model:

$$k_x = k_y = \frac{-R^2}{8(1-\epsilon)}\left[\ln(1-\epsilon) + \epsilon + \frac{\epsilon^2}{2}\right] \tag{6}$$

with k_z again being given by Equation (5). The characteristic D.O.A. for fluid flow is defined by k_x/k_z. Thus, from Equations (4) to (6)

$$(k_x/k_z)_{\text{Happel}} = \frac{\ln(1-\epsilon) + \dfrac{\epsilon(2-\epsilon)}{\epsilon^2 - 2\epsilon + 2}}{2\ln(1-\epsilon) + 2\epsilon + \epsilon^2} \tag{7}$$

$$(k_x/k_z)_{\text{Kuwabara}} = \tfrac{1}{2} \tag{8}$$

The above two predictions for k_x/k_z are displayed in Figure 2. It is appropriate to mention here that the fluid flow predictions [Equations (4) to (8)] are valid only for monosized cylinders, whereas the diffusion predictions [Equations (1) to (3)] are valid for any arbitrary size distribution.

It is interesting to note that the hydrodynamic D.O.A., k_x/k_z, is independent of the porosity ϵ according to the Kuwabara model [Equation (8)]. Since k_z is given by Equation (5) for both the Happel and Kuwabara models, it is apparent that the dotted lines in Figure 2 compare directly not only the D.O.A. but also the k_x predictions of both models. However, the experimental data of Kirsch and Fuchs (1967) indicates that the Kuwabara model provides the better prediction for k_x in practice.

An important difference between the D.O.A.'s for diffusion and fluid flow is evident from Figure 2, namely, that for $\epsilon \to 1$, $D_x/D_z \to 1$ while $k_x/k_z \to 0.5$. The inference is that very dilute systems of parallel circular cylinders behave isotropically with respect to diffusion (or electrical conduction) but very anisotropically with

* The predictions presented in this work for diffusion are valid also for electrical conduction through liquid saturated media composed of nonconducting particles (upon identification of the diffusivity D with the electrical conductivity), provided surface conductance effects may be neglected. This surface conductance (zeta potential) effect is generally only important when we deal with very dilute electrolytes (Neale and Nader, 1976; Levine et al., 1975).

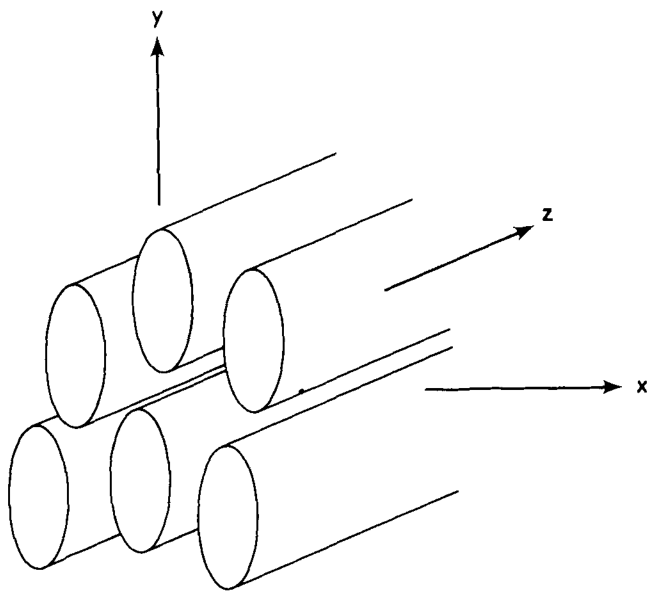

Fig. 3. A bank of parallel monosized elliptic cylinders.

respect to fluid flow. As $\epsilon \to 0$, $D_x/D_z \to 0.5$ and $k_x/k_z \to 0.5$, although these limits are essentially hypothetical for monosized cylinders (for which the minimum attainable porosity is 0.0931, corresponding to an equilateral-triangular array). For intermediate porosities it is evident that the D.O.A.'s for diffusion and fluid flow (Kuwabara model) can differ appreciably. It should be noted that the hydrodynamic D.O.A. predictions above pertain specifically to creeping Newtonian fluid flow (it would be very interesting to measure the effects of non-Newtonian behavior and Reynolds number on k_x/k_z).

The significant conclusions reached in the exploratory study summarized above induced the author to study the fundamentally more important anisotropic system of a cluster of parallel elliptic cylindrical fibers.

POROUS MEDIA COMPOSED OF PARALLEL ELLIPTIC CYLINDRICAL FIBERS

Figure 3 depicts a cluster of parallel (aligned) monosized elliptic cylindrical fibers distributed in a random array. The x direction is taken to be parallel to the minor axis of the elliptical cross section of each of the aligned fibers. The shape of each elliptical cross section is characterized by its eccentricity E (E = length of minor axis/length of major axis). In this study, in contrast to the preceding study involving circular cylinders, attention will be focused on diffusion and fluid flow occurring in the principal x and y directions only.

There do not appear to be available in the literature any predictions for diffusion (or electrical conduction) through a cluster of elliptic cylindrical fibers. However, the generalized geometric model developed earlier to study transport processes within porous media composed of spherical particles (Neale and Nader, 1973, 1974) or circular cylinders (Neale and Masliyah, 1975), and successfully applied in a recent study of diffusion (electrical conduction) through anisotropic porous media composed of aligned spheroidal particles (Neale and Nader, 1976), provides a simple method of calculating the effective diffusivities in the principal directions of the present anisotropic system of parallel elliptic cylinders (see Appendix for theoretical analysis). Thus

$$\lambda_x = D_x/D = T(T - E)/(1 - TE) \quad (9)$$

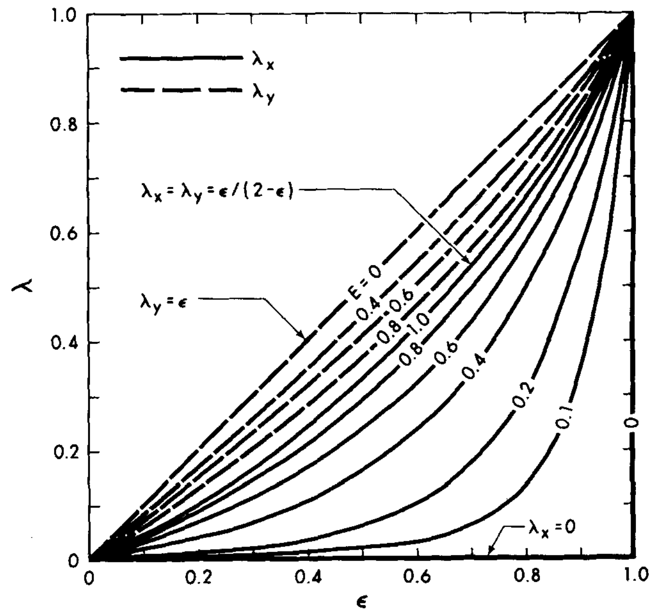

Fig. 4. Predictions for λ_x [Equation (9)] and λ_y [Equation (10)] for parallel elliptic cylinders.

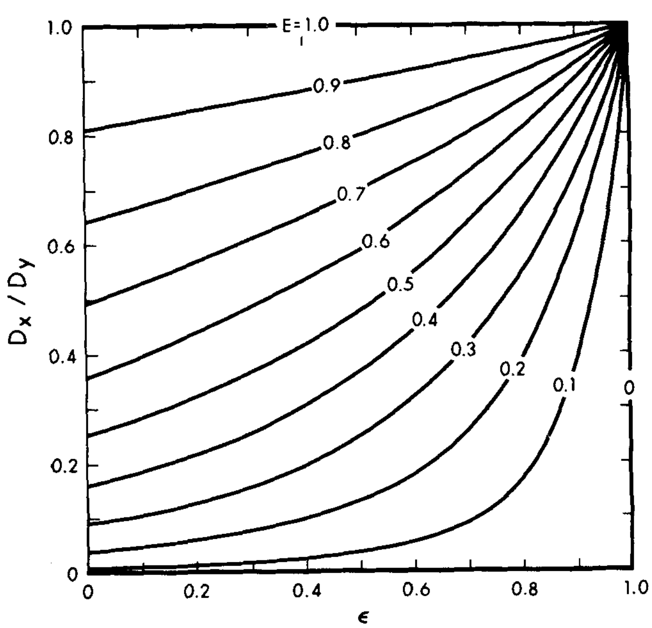

Fig. 5. Predictions for D_x/D_y [Equation (13] for parallel elliptic cylinders.

$$\lambda_y = D_y/D = (T - E)/(T - T^2E) \quad (10)$$

$$\lambda_z = D_z/D = \epsilon \quad (11)$$

where the λ's denote dimensionless diffusivity factors (Neale and Nader, 1973, 1976), and T is defined by

$$T = [\sqrt{(1-\epsilon)^2(1-E^2)^2 + 4E^2} - (1-\epsilon)(1-E^2)]/2E \quad (12)$$

(physically, T represents the eccentricity of the outer envelope of the unit cell in the geometric model depicted in Figure 7).

The predictions of Equations (9) and (10) are displayed in Figure 4 as functions of ϵ for various values of E. Equation (11) actually represents the exact solution for all physically attainable values of ϵ. Unfortunately, there are no experimental diffusion (conduction) data available in the literature for elliptic cylinders with

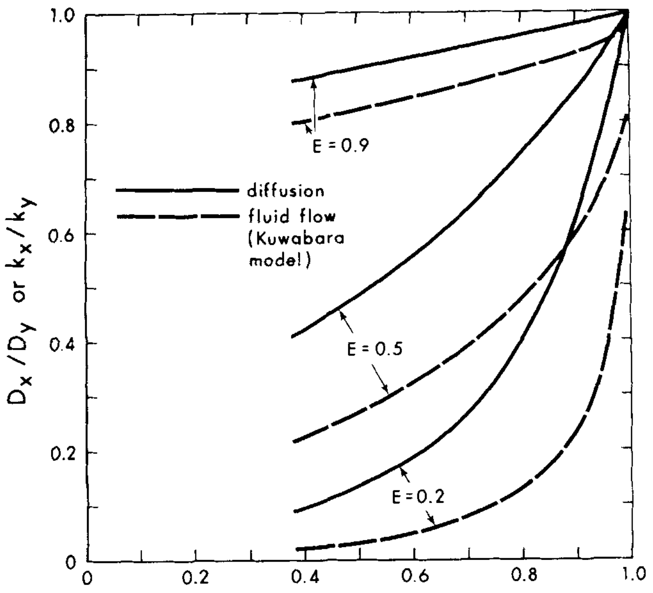

Fig. 6. Comparison between D_x/D_y and k_x/k_y for parallel mono-sized elliptic cylinders. ———, present work [Equation (13)]; — — — —, Epstein and Masliyah (1972).

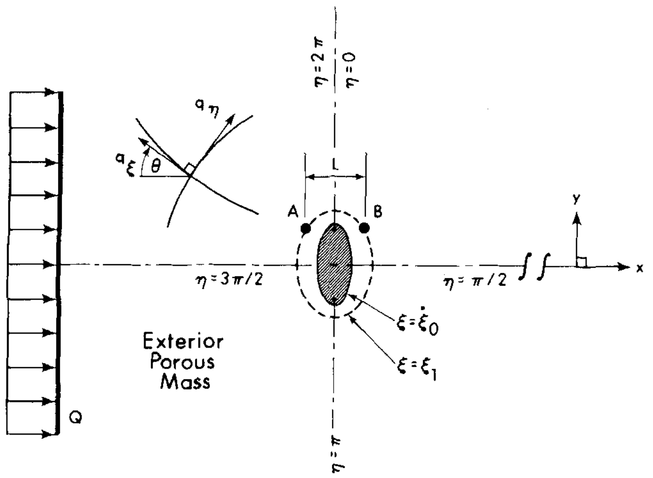

Fig. 7. The proposed model for a bank of parallel elliptic cylinders (cross section).

which to compare the predictions (9) and (10). However, these predictions have been obtained from a geometric model which has already proved itself by generating predictions which are in good agreement with experimental data for diffusion through the closely related systems of spherical particles (Neale and Nader, 1973) as well as circular cylindrical and spheroidal particles (Neale and Nader, 1976).

The D.O.A. for diffusion *within* the xy plane is defined as D_x/D_y (≤ 1) which, from Equations (9) and (10), is given by

$$D_x/D_y = T^2 \quad (13)$$

To completely characterize the three-dimensional anisotropy of the system, it would suffice to determine the additional D.O.A.'s: $D_x/D_z = \lambda_x/\epsilon$ and $D_y/D_z = \lambda_y/\epsilon$.) The values of D_x/D_y predicted by Equation (13) are displayed in Figure 5 as functions of ϵ and E. The following limiting values are of particular interest:

as $E \to 1$ (long circular cylinders), $D_x/D_y \to 1$ for all ϵ

as $E \to 0$ (long thin rectangular plates), $D_x/D_y \to 0$ for all ϵ

as $\epsilon \to 1$ (dilute systems), $D_x/D_y \to 1$ for all E

as $\epsilon \to 0$ (concentrated systems), $D_x/D_y \to E^2$

The first three limits actually represent exact solutions to the problem at hand.

The case of creeping Newtonian fluid flow through clusters of identical parallel (aligned) elliptic cylinders has already been treated by Epstein and Masliyah (1972) using the Happel and Kuwabara cell models. Predictions for the Kozeny constant K were presented for flow in the x and y directions but not for the z direction, for the particular geometries characterized by $E = 0.9$, 0.5, and 0.2. Their predictions were indicated to be consistent with experimental data available in the literature for thick mats of flattened nylon fibers. Numerical solution of the Navier-Stokes equation was necessary because no closed form solution of this equation for the cell model geometry (or even for a single elliptic cylinder in unbounded space) is known. Thus, since $K \propto 1/k$, the results of Epstein and Masliyah may be used to determine values of the D.O.A. k_x/k_y for creeping fluid flow. These predictions, based on Kuwabara's model [which is considered more suitable than Happel's model for the case of parallel cylinders (Kirsch and Fuchs, 1967)], are depicted in Figure 6, together with the corresponding predictions of the D.O.A. for diffusion calculated from Equation (13).

It is apparent that for a given system (that is, for given values of E and ϵ) the D.O.A. for diffusion is always closer to unity than that for fluid flow, indicating that clusters of elliptic cylinders are more anisotropic (within the xy plane) with respect to fluid flow than with respect to diffusion. Moreover, this effect becomes increasingly pronounced as E decreases, that is, as the fibers become flatter. Equally important, Figures 2 and 6 would appear to indicate that no simple general analogy exists between diffusion (electrical conduction) and fluid flow occurring within anisotropic porous media. However, it should be noted that the limited data presented in Figure 6 do Wyllie and Spangler (1952) that D.O.A.'s for fluid flow conform reasonably closely to the semiempirical rule of are proportional to the square of the corresponding D.O.A.'s for electrical conduction. The data in Figure 2 disagree with this rule.

OTHER ANISOTROPIC POROUS MEDIA

Although only systems of parallel fibers have been examined in this study, it seems likely that qualitatively similar D.O.A. predictions would be obtained for other types of anisotropic porous media encountered in practice, for example, for beds of sedimented disclike or needlelike particles. Actually, the problem of diffusion (electrical conduction) through a bed of oblate or prolate spheroids, approximating disclike and needlelike particles, respectively, aligned with their axes of revolution collinear with the x direction (Figure 7) has already been treated analytically by Neale and Nader (1976) using the same geometric model as in the present work. Predictions for D_x and D_y ($= D_z$) were presented so the D.O.A. for diffusion D_x/D_y may be determined directly. (Incidentally, these D.O.A.'s exhibit trends very similar to those presented in Figure 5 for elliptic cylindrical fibers.) The problem of creeping Newtonian fluid flow through beds of aligned spheroidal particles has been solved numerically by Epstein and Maslivah (1972), employing the cell models of Happel and Kuwabara. However, these workers only performed calculations for flow in the axially symmetric x direction, and not for the y direction, so the D.O.A. for fluid flow k_x/k_y cannot be computed for comparison with the D.O.A. for diffusion derived here.

The work reported here has been exclusively concerned with homogeneous systems. The case of nonhomogeneous (nonuniform porosity) anisotropic porous media is definitely of practical importance, especially in relation to underground rock formations, but such systems are less easily defined mathematically and would consequently require a more sophisticated modeling technique than that used here.

DISCUSSION

The presented results establish quite clearly that the D.O.A. of any arbitrary anisotropic porous medium is definitely a function of the particular transport process being studied, as would be expected on intuitive grounds alone. This conclusion has been reached by studying two distinct well-defined transport processes, namely, creeping Newtonian fluid flow and simple diffusion (electrical conduction). Perhaps of most practical significance is the fact that there does not appear to be any simple general analogy between fluid flow and diffusion occurring in anisotropic porous media (in particular, there is no simple relationship between the D.O.A.'s for these two transport processes). Other transport processes, such as heat transfer and sound transmission, would presumably be associated with D.O.A.'s different from those calculated in this work for fluid flow and diffusion. These processes would be more difficult to study since transmission (of sound or heat) would occur through the solid particles as well as through the saturating fluid. In addition, natural convection effects might be important in practical situations involving heat transfer through anisotropic porous media (for example, during fire flooding of petroleum reservoirs).

The present work (Figure 6) demonstrates that it is not judicious to assume that the D.O.A.'s for any two arbitrary transport processes are equal, or even nearly equal, and indicates that the quantity degree of anisotropy is intrinsically an ambiguous parameter, unless it is qualified by reference to a specific transport process.

It is appropriate to mention here the radically new capillary type of model of porous media conceived by Dullien (1975*a*). Basically, if the pore size distribution and porosity of an isotropic porous medium are measured, then the Dullien model can be used to predict the macroscopic transport properties (permeability and effective diffusivity) directly, without having to introduce any uncertainty factors such as the notorious tortuosity. The principal significance of the Dullien model is that it serves to elucidate the complex interdependence between permeability (diffusivity) and pore space structure. Of principal interest in the present context is the claim (Dullien, 1975*b*) that the model can be generalized to simulate anisotropic porous media. It would be instructive to compare the present D.O.A. predictions with predictions based upon the Dullien model.

It is hoped that the present introductory study will induce further research into the practically important, but hitherto rather neglected, field of transport processes occurring within anisotropic porous media.

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NOTATION

- a = distance between focus and geometric center of ellipse
- c = concentration of diffusing species
- c_A, c_B = concentrations at points A and B on outer envelope of unit cell
- c_o = constant reference concentration (at $\eta = 0$)
- D = absolute diffusivity of diffusing species (in absence of porous medium)
- D_i = observed macroscopic diffusivity in presence of porous medium ($i = x, y, z$)
- E = eccentricity of ellipse (= length of minor axis/length of major axis) = $\tanh \xi_0$
- L = distance between points A and B in modeled system
- k_i = permeability of porous medium ($i = x, y, z$)
- R = radius of circular cylinder
- T = $\tanh \xi_1$ (= eccentricity of outer envelope of unit cell in Figure 7)
- q_ξ = flux component in direction normal to surfaces of constant ξ
- Q = magnitude of uniform mainstream diffusive flux
- $[x,y,z]$ = Cartesian coordinates

Greek Letters

- ϵ = porosity (void fraction) of porous medium
- λ_i = diffusivity factor ($= D_i/D$), $i = x, y, z$
- $[\xi,\eta]$ = elliptic coordinates
- θ = angle between direction of q_ξ and the negative x direction
- ξ_0 = coordinate representation of elliptic cylinder in modeled system
- ξ_1 = coordinate representation of outer envelope of unit cell

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APPENDIX

The geometric model (Figure 7) and theory for two-dimensional diffusion through a cluster of aligned elliptic cylinders are completely analogous to those for a bed of aligned spheroidal particles (Neale and Nader, 1976). Consequently, the present theory is summarized only very briefly here. Elliptic coordinates $[\xi, \eta]$ which are related to Cartesian coordinates by

$$x = a \sinh \xi \sin\eta \tag{A1}$$

$$y = a \cosh \xi \cos\eta \tag{A2}$$

are employed (where $0 \leq \xi < \infty$, $0 \leq \eta < 2\pi$).

For diffusion in the x direction, a solution of Laplace's equation

$$\nabla^2 c = 0 \tag{A3}$$

is sought which satisfies the following boundary conditions:

$$\text{at} \quad \xi = \xi_0, \quad q_\xi = 0 \tag{A4}$$

$$\text{at} \quad \xi = \xi_1, \quad q_\xi = -Q \cos\theta \tag{A5}$$

where q_ξ is given by the appropriate form of Fick's law; namely

$$q_\xi = -D \frac{1}{a\sqrt{\sinh^2 \xi + \sin^2 \eta}} \frac{\partial c}{\partial \xi} \tag{A6}$$

and, from basic geometry

$$\cos\theta = -\frac{\cosh \xi \sin \eta}{\sqrt{\sinh^2 \xi + \sin^2 \eta}} \tag{A7}$$

General solutions of Equation (A3) in elliptic coordinates are available in the literature (Moon and Spencer, 1961). The particular solution which satisfies Equations (A3) to (A7) may be verified to be

$$c = c_o - (Qa/D) \left[\frac{\cosh \xi - \tanh \xi_0 \sinh \xi}{\tanh \xi_1 - \tanh \xi_0} \right] \sin\eta \tag{A8}$$

The effective diffusivity D_x is determined (Neale and Nader, 1976) by applying the macroscopic form of Fick's law to the unit cell depicted in Figure 7; namely

$$Q = D_x(c_A - c_B)/L \tag{A9}$$

where $L = 2a \sinh \xi_1 \sin\eta$. Combining Equations (A9) and (A8), we get the sought prediction for D_x, which is presented in Equation (9) of the main text (note that $E = \tanh \xi_0$ and $T = \tanh \xi_1$). The porosity of the unit cell must be equal to the porosity ϵ of the original system (Neale and Nader, 1976) in order to ensure macroscopic homogeneity of the modelled system; thus

$$1 - \epsilon = \frac{\sinh \xi_0 \cosh \xi_0}{\sinh \xi_1 \cosh \xi_1} \tag{A11}$$

The hyperbolic terms in this equation may be expressed solely in terms of E and T. Solving the resulting equation analytically for T then, we get Equation (12) of the main text.

A similar set of calculations can be performed for the case of diffusion in the y direction, yielding the prediction for D_y/D presented in Equation (10). In reality, however, it is far more convenient to make avail of a standard mathematical transformation. Thus, it may be verified that Equations (A8) and (9) become valid for flow in the y direction upon replacing each $\sinh\xi$ term by $\cosh\xi$ and each $\cosh\xi$ term by $\sinh\xi$ (this may be interpreted physically as an interchange of any minor axis, of length $2a \sinh\xi$, with the corresponding major axis, of length $2a \cosh\xi$, and vice versa).

The prediction for D_z [Equation (11)] follows without proof, since the cross-sectional area available for diffusion does not change in the z direction. (The results for diffusion developed here are actually independent of the size distribution for clusters of aligned cylinders having equal eccentricity.)

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Thermodynamics of the Sulfur Dioxide-Seawater System

ABDUL H. ABDULSATTAR
SRINIVASAN SRIDHAR
and
LEROY A. BROMLEY
Department of Chemical Engineering
University of California
Berkeley, California 94720

The solubility of sulfur dioxide in seawater in the range 10° to 25°C and 10^{-5} to 1 molal is predicted based on a simplified chemical model. Estimates are also given of the resulting *pH* and the attendant distribution of sulfur and carbon dioxide species. Sulfur dioxide is considerably more soluble in seawater or other naturally alkaline waters than in pure water.

SCOPE

A novel pretreatment process for desalination that softens seawater and reduces corrosion has been proposed independently by Roy and Yahalom (1968) and Bromley (1969). In this process, sulfur dioxide from power plant flue gases or other inexpensive sources is injected directly into the seawater in a sparger, packed or a spray tower. This process is economical, permits better *pH* control

Abdul H. Abdulsattar is with the Bechtel Corporation, P.O. Box 3965, San Francisco, California 94119. Srinivasan Sridhar is with Union Carbide Technical Center, P.O. Box 8361, South Charleston, West Virginia 25303.